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M-brane intersections from worldvolume superalgebras

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Abstract

It is known that the M-branes of M-theory correspond to p -form charges in the $D = 11$ spacetime supersymmetry algebra. Here we show that their intersections are encoded in the p -form charges of their *worldvolume* supersymmetry algebras. Triple intersections are encoded in double intersection worldvolume algebras with eight supercharges. © 1998 Elsevier Science B.V.

1. Introduction

The 11-dimensional ($D = 11$) M-theory is currently the leading candidate for a unified theory of particle interactions, despite the fact that a complete understanding of its microscopic degrees of freedom (especially those relevant to compactifications to four spacetime dimensions) remains elusive. Fortunately, many of the implications of M-theory can be deduced from semi-classical considerations. At this level, M-theory appears to be a theory of one ‘basic’, or $1/2$ -supersymmetric, ‘object’ which can appear in various dual forms. These can be found as solutions of the $D = 11$ supergravity field equations. Alternatively, the possibilities may be investigated more abstractly via the ‘M-theory superalgebra’ [1]; this is a modification of the standard $D = 11$ supersymme-

try algebra to incorporate 2-form and 5-form central charges [2]. The space components of these charges are associated [3,4] with the M-2-brane and M-5-brane. The time components of the 2-form charge may be associated [4,5] with a boundary of the $D = 11$ spacetime, as in the Hořava-Witten (HW) construction of the heterotic string [6]. For terminological convenience we refer to this as the M-9-brane; it can in fact be viewed as a decompactification limit of eight D-8-branes coincident with an orientifold 8-plane in Type IA superstring theory [7]. The time component of the 5-form charge is associated with the Kaluza-Klein (KK) monopole solution of $D = 11$ supergravity, alias the D-6-brane of IIA supergravity [8]. We shall refer to this as the M-KK-monopole. Finally, the space components of the 11-momentum are associated with the massless quanta of $D = 11$ supergravity or, at the classical level, with the ‘M-wave’ solution of $D = 11$ supergravity [9]. This is the only $1/2$ supersymmetric configuration permitted by the standard $D = 11$ supersymmetry algebra,

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hence the necessity of considering the full M-theory superalgebra.

To summarize, there are five ‘basic’, 1/2-supersymmetric, constituents of M-theory. These are M-Wave, M-2-brane, M-5-brane, M-KK-monopole, M-9-brane.

(1)

The M-2-brane and M-5-brane are often referred to collectively as ‘M-branes’; they are in fact the only *bone fide* branes. All the same, we shall find it convenient to refer collectively to all five ‘basic’ constituents as ‘M-branes’. M-theory configurations with less than 1/2 supersymmetry may now be found as ‘intersecting’ M-branes [10]. The simplest cases are those in which two M-branes intersect to yield a configuration with 1/4 supersymmetry (of the M-theory vacuum). From the point of view of each of the two constituent branes, the intersection appears as a 1/2 supersymmetric object in its worldvolume. Thus, one way to study 1/4-supersymmetric intersections is by looking for 1/2 supersymmetric solutions of the *worldvolume* field equations of the various M-branes. For example, it is known that two M-5-branes may intersect on a 3-brane, preserving 1/4 supersymmetry [10,11], and that an M-2-brane may end on an M-5-brane [12,13], again preserving 1/4 supersymmetry (the boundary may be considered as a string intersection for present purposes). From the perspective of the M-5-brane (or one of them if there is more than one) these intersections should appear as 1/2 supersymmetric 3-brane and string solutions of its effective worldvolume field equations. Both solutions have recently been found [14,15], following the discovery of analogous solutions of the worldvolume field equations of D-branes [16,17]. It is remarkable that these solutions, while ostensibly just configurations of a particular worldvolume field theory, actually provide their own *spacetime* interpretation as intersecting branes; this comes about because the worldvolume scalars determine the spacetime embedding.

Here we take this a step further by showing that the *spacetime* interpretation of a worldvolume p -brane preserving 1/2 supersymmetry is encoded in the *worldvolume* supersymmetry algebra. Indeed, all 1/4 supersymmetric M-brane intersections may be deduced this way. The first point to appreciate is that worldvolume p -branes are associated with p -forms

in the worldvolume supersymmetry algebra. In itself, this is not surprising given our experience with the spacetime supersymmetry algebra. For example, it was pointed out in [15] that the maximal central extension of the $D = 6$ (2,0) worldvolume supertranslation algebra of the M-5-brane has the anticommutator

$$\{Q_\alpha^I, Q_\beta^J\} = \Omega^{IJ} P_{[\alpha\beta]} + Y_{[\alpha\beta]}^{[IJ]} + Z_{(\alpha\beta)}^{(IJ)},$$

$$\Omega_{IJ} Y^{[IJ]} = 0, \quad (2)$$

where $\alpha, \beta = 1, \dots, 4$ is an index of $SU^*(4) \cong Spin(5,1)$ and $I = 1, \dots, 4$ is an index of $Sp(2)$, with Ω^{IJ} being its invariant antisymmetric tensor. The Y -charge is a worldvolume 1-form and the Z -charge a worldvolume self-dual 3-form. The presence of such charges in the worldvolume supersymmetry algebra is implied by the existence of the 1/2 supersymmetric string and 3-brane solutions of the M-5-brane worldvolume field equations. There is still a puzzle to be resolved, however. There are a total of five 1-form charges and ten 3-form charges (in the irreducible **5** and **10** representations of $Sp(2)$), which exceeds the number of distinct worldvolume strings and 3-branes. In the spacetime supersymmetry algebra, every p -form charge corresponds to a distinct p -brane. This is clearly not so for worldvolume algebras. The resolution of this puzzle is simply that the $Sp(2)$ representations provide the information needed to reconstruct the *spacetime* interpretation of the branes within the M-5-brane. For example, identifying $sp(2) \cong Spin(5)$ as the double cover of the $SO(5)$ rotation group transverse to the M-5-brane worldvolume in the $D = 11$ spacetime, we see that the 1-forms Y define a vector in this transverse 5-space. This information serves to identify the 1-brane in the worldvolume as the boundary of an M-2-brane in the M-5-brane. Note, however, that the **5** representation of $Sp(2)$ could equally well have been interpreted as defining a 4-form in the transverse 5-space, so the 1-brane has another spacetime interpretation as the intersection of the M-5-brane with another M-5-brane, which is again an M-brane intersection known to preserve 1/4 supersymmetry [18].

So far we have implicitly considered only the space components of Y . As pointed out in [15], the time components can be viewed as the space compo-

nents of a worldvolume dual 5-form and are therefore charges for a space-filling 5-brane in the M-5-brane. We interpret this to mean that the given M-5-brane is actually inside another brane, so that its ‘intersection’ with it coincides with the M-5-brane. If the **5** representation is taken to define a 1-form in transverse 5-space we might then conclude that the M-5-brane is inside a 6-brane. There is no M-theory 6-brane as such but there is the M-KK-monopole. Adopting this interpretation we conclude, correctly [19], that an M-5-brane ‘inside’ an M-KK-monopole preserves 1/4 supersymmetry. If instead we take the **5** representation to define a transverse 4-form then we conclude, again correctly [20], that an M-5-brane inside an M-9-brane preserves 1/4 supersymmetry; in fact, this is just the heterotic 5-brane in the HW-formulation of the heterotic string theory.

There is still much to learn from the algebra (Eq. (2)). For example, we have not yet considered the Z charges in the **10** representation of $Sp(2)$. Suffice it to say that the complete analysis precisely duplicates the known classification of 1/4 supersymmetric intersections involving an M-5-brane. There remains a question of consistency. To take an example, we have just deduced from the M-5-brane worldvolume superalgebra that it can have a 1/4-supersymmetric intersection with an M-2-brane. We should be able to see this from the M-2-brane’s worldvolume algebra. This is (after gauge fixing) a $D = 3$ field theory with $N = 8$ supersymmetry. Allowing for all possible p -form central charges in the $N = 8$ $D = 3$ supertranslation algebra we arrive at the anticommutator

$$\{Q_{\alpha}^i, Q_{\beta}^j\} = \delta^{ij} P_{(\alpha\beta)} + Z_{(\alpha\beta)}^{(ij)} + \varepsilon_{\alpha\beta} Z^{[ij]},$$

$$\delta_{ij} Z^{(ij)} = 0, \quad (3)$$

where Q^i are the eight $D = 3$ Majorana spinor supercharges ($i = 1, \dots, 8$), and P is the 3-momentum². This supersymmetry algebra has an $SO(8)$ automorphism group, which we interpret as the rotation group in the transverse 8-space. The

supercharges do not transform as an 8-vector, however, but as a chiral $SO(8)$ spinor. The 0-form and 1-form central charges are therefore

$$Z(0\text{-form}): \mathbf{28}, \quad Z(1\text{-form}): \mathbf{35}^+, \quad (4)$$

where the **28** is either a 2-form or 6-form of the transverse 8-space and the **35**⁺ is a self-dual transverse 4-form. Consider first the worldvolume 0-forms, corresponding to a 0-brane on the $D = 3$ worldvolume. Choosing the 2-form interpretation of the **28** representation we see that the 0-brane is the result of an intersection with another M-2-brane, a known 1/4 supersymmetric M-brane intersection [10]. If we instead choose the 6-form interpretation of the **28** then the worldvolume 0-brane acquires the interpretation as the intersection of the M-2-brane with a KK-monopole [21]. Consider now the worldvolume 1-forms, corresponding to a 1-brane on the $D = 3$ worldvolume. This 1-brane is determined by a 4-plane in the transverse space and is therefore the intersection of an M-5-brane with the M-2-brane. We therefore recover from the M-2-brane worldvolume superalgebra the intersection with the M-5-brane that we previously deduced from the latter’s worldvolume superalgebra. Again, there is much more to be learnt from the algebra (Eq. (3)), but we postpone the complete analysis.

The remaining M-brane superalgebras can be dealt with in the same way. The final result is that each 1/4-supersymmetric intersection of two M-branes occurs precisely once in each of the lists of intersections derived from the worldvolume supersymmetry algebras of its constituents. If the same analysis is applied to D-branes the results are not quite so nice. It remains true that every intersecting brane configuration preserving 1/4 supersymmetry corresponds to some spacetime interpretation of a p -form charge in the worldvolume superalgebra of either participating brane. Conversely, every p -form in a worldvolume superalgebra has a spacetime interpretation as a 1/4-supersymmetric configuration of intersecting branes, but it is no longer always the case that *every* such interpretation corresponds to a 1/4-supersymmetric configuration; one has to impose additional ad hoc interpretational rules. The reason for this can be traced to some peculiarities of supersymmetry alge-

² This, at least, is the obvious interpretation of this ‘charge’. We shall see later that it has another interpretation.

bras with 16 supercharges. The complete list of such algebras is as follows:

	$D = 1$	$N = 16$	$so(9)$	16	D-0
*	$D = 2$	(16,0)	$so(9)$	16	M-W
	$D = 2$	(16,0)	$so(8)$	$\mathbf{8}_s \oplus \mathbf{8}_s$	IIB-W
	$D = 2$	(8,8)	$so(8)$	$\mathbf{8}_s \oplus \mathbf{8}_c$	IIB-1
	$D = 2$	(8,8)	$so(8)$	$\mathbf{8}_s \oplus \mathbf{8}_s$	IIA-1
*	$D = 3$	$N = 8$	$so(8)$	8_s	M-2
	$D = 4$	$N = 4$	$so(6)$	$\mathbf{4} \oplus \bar{\mathbf{4}}$	D-3
	$D = 5$	$N = 2$	$so(5)$	4	D-4
*	$D = 6$	(2,0)	$so(5)$	4	M-5
	$D = 6$	(1,1)	$so(4)$	4	IIB-5
*	$D = 7$	$N = 1$	$so(3)$	2	D-6, M-KK
	$D = 8$	$N = 1$	$so(2)$	2	D-7
	$D = 9$	$N = 1$	Z_2	—	D-8
*	$D = 10$	(1,0)	—	—	D-9, M-9

The value of N is given in the third column, with the convention that minimal supersymmetry is always $N = 1$; for $D = 2, 6, 10$, the number of chiral and antichiral spinors must be specified separately. The automorphism, or ‘R-symmetry’, algebra is given in the fourth column; the group is generally the spinorial covering group and the representation to which the supercharges belong is given in the fifth column. For $D = 1, 2$ we have not given the largest possible automorphism algebra. For example, the (8,8) algebras actually have a $Spin(8)$ group acting independently on spinors of each chirality, but in the string-theory realization only the diagonal $Spin(8)$ survives. Similarly, in $D = 1$ the maximal automorphism group is $SO(16)$ but only $SO(9)$ is realized by 0-branes. This is because the supercharges must be spinors of the R-symmetry group as well as of the worldvolume Lorentz group, and $SO(9)$ is the largest subgroup of $SO(16)$ with a 16-dimensional spinor representation. In the last column we give a brane field theory that realizes the supersymmetry algebra. Some branes do not appear in the list because they realize only a proper subgroup of the automorphism algebra as a consequence of being the dimensional reduction of another brane with the same algebra. For example, the IIA-wave is essentially the same as the M-wave but with the $SO(9)$ symmetry reduced to $SO(8)$.

If one associates the $so(n)$ R-symmetry algebra of each of the above supersymmetry algebras to an $SO(n)$ rotation group in a space transverse to a $(p + 1)$ -dimensional worldvolume then one arrives at

an interpretation in which the p -brane is embedded in a larger spacetime of total dimension $D = (n + p + 1)$. This total dimension is in each case either $D = 10$ or $D = 11$. This fact might well be considered remarkable to someone studying these algebras as a mathematical exercise, but it is of course expected from their interpretation as worldvolume superalgebras of $D = 10$ or $D = 11$ branes. Note that a single transverse dimension would be associated with a trivial $SO(1)$ rotation group so that in those cases for which the R-symmetry algebra is trivial there is an ambiguity in the dimension of the transverse space: it could be zero or one. For example, if we suppose that the transverse space associated to the $D = 10$ (1,0) algebra has dimension one we may interpret the algebra as that of the M-9-brane. If instead we choose the transverse space to have dimension zero we may interpret the algebra as that of the D-9-brane, which is effectively the $D = 10$ spacetime of the Type I superstring. The $D = 9$ $N = 1$ supersymmetry algebra is another rather special case. Its Z_2 automorphism group is the vestige of a rotation group in a 1-dimensional transverse space, so the D-8-brane acquires a natural $D = 10$ interpretation but, as usual, its 11-dimensional interpretation is obscure.

An inspection of the above supersymmetry algebras reveals that most are dimensional reductions of some other one. The only exceptions are the five supersymmetry algebras corresponding to the worldvolume field theories of the five M-branes. These have been marked with an asterisk in the first column. The M-wave, M-5-brane and M-9-brane algebras are manifestly not reductions of any other algebra in the list. Dimensional reduction of the D-3-brane algebra yields an $N = 8$ $D = 3$ algebra but its automorphism algebra is $so(7)$ rather than $so(8)$. This is of course the algebra of the D-2-brane, which we did not include because it is just the reduction of the M-2-brane. Thus, the M-2-brane algebra is also not the reduction of any other one. This leaves the algebra associated to the M-KK-monopole. Strictly speaking, this is an exception to the rule because it is also the D-6-brane algebra, which is the reduction of the D-7-brane algebra. However, this algebra shares the nice features of the other M-brane algebras, and when one takes into account the hidden dimension implicit in the KK-monopole metric, there is a natu-

ral $D = 11$ interpretation. In any case, its inclusion among the ‘exceptional’ cases is forced by the intersecting brane interpretation of the p -form charges in the other M-brane algebras. We learn from all this that when determining the possible $1/4$ -supersymmetric brane intersections from the worldvolume supersymmetry algebras of participating branes we need only consider the five M-branes. Thus, the worldvolume superalgebras of $D = 10$ and $D = 11$ branes proclaim the ‘supremacy’ of M-theory.

Having determined all $1/4$ -supersymmetric intersections of M-branes from the supersymmetry algebras with 16 supercharges one is led to wonder whether intersections preserving less supersymmetry are encoded in the supersymmetry algebras with fewer supersymmetry charges. The answer appears to be a qualified ‘yes’, but we shall not attempt a systematic analysis here. Rather, we shall present a few illustrative cases in which triple intersections with $1/8$ supersymmetry can be deduced from the p -form charges in the superalgebra with 8 supercharges corresponding to $1/4$ -supersymmetric pairwise brane intersections. One case of interest is Type I brane intersections viewed as triple intersections in which the ‘third brane’ is a D-9-brane. Another class of cases can be deduced from the worldvolume algebra of the 3-brane in the M-theory 5-brane. Analysis of the p -forms in this algebra shows, in particular, that the 3-brane in the M-5-brane is a D-brane for the self-dual string. We shall provide both $D = 10$ IIB and $D = 11$ M-theory explanations of this fact.

2. M-brane intersections

2.1. M-wave

The M-wave algebra is

$$\{Q_-^i, Q_-^j\} = \delta_{ij} P_- + Z_-^{(ij)}, \quad \delta_{ij} Z_-^{(ij)} = 0, \quad (5)$$

where the supercharges Q_-^i ($i = 1, \dots, 16$) are in the **16** spinor representation of $SO(9)$. This has the obvious reduction to the $D = 10$ D-0-brane algebra

$$\{Q^i, Q^j\} = \delta_{ij} H + Z^{(ij)} (\mathbf{126} \oplus \mathbf{9}). \quad (6)$$

As indicated, the 0-form central charges in this algebra fall into the **126** and **9** representations of

$SO(9)$, as do the $D = 2$ self-dual vector Z-charges in the M-wave algebra. Intersections of M-waves preserving $1/4$ supersymmetry have been classified in [21], but since these are less familiar than those involving D-0-branes it is helpful to consider the latter first. The results for the M-wave are then obtained by re-interpreting the D-0-brane results in $D = 11$.

The **126** of $SO(9)$ is either a 4-form or a 5-form in the transverse 9-space. This corresponds to a D-0-brane in a D-4-brane or a KK-monopole, respectively. The **9** of $SO(9)$ is either a 1-form or an 8-form in the transverse 9-space. This corresponds to a D-0-brane in a fundamental string or a D-8-brane, respectively. The $D = 11$ interpretation of the same configurations is, in the notation of [21],

$$(1|W, M5), \quad (1|W, KK), \quad (1|W, M2), \\ (1|W, M9). \quad (7)$$

This is the complete list of $1/4$ -supersymmetric intersections of M-waves.

2.2. M-2-brane

The M-2-brane algebra is given in (Eq. (3)). We considered earlier the spacetime interpretation of the space components of the 1-form and 3-form charges. This led to the following M2-brane intersections:

$$(0|M2, M2), \quad (0|M2, KK), \quad (1|M2, M5). \quad (8)$$

The time components of the 1-form charges are equivalent to space components of the dual 2-forms and hence correspond to a ‘worldvolume 2-brane’, i.e. worldvolume filling $1/2$ -supersymmetric 2-branes. These cases are naturally interpreted as ‘intersections’ in which the original M-2-brane is inside some larger brane. The latter is nominally a 6-brane since it is determined by an additional 4-plane in the transverse space but, as earlier, this is to be interpreted as a KK-monopole. We thus conclude that an M-2-brane inside a KK-monopole preserves $1/4$ supersymmetry, as indeed it does. This yields the intersection

$$(2|M2, KK). \quad (9)$$

To find the remaining intersections of M-2-branes we must take into account that the algebra (Eq. (3)) admits $1/2$ supersymmetric charge configurations in

which only the 3-vector P is non-zero; it must then be null. Since P is also a singlet of $SO(8)$ we may consider it to be either a 0-form in the transverse 8-space or an 8-form. In the former case P has the obvious interpretation as the null 3-momentum of an M-wave ‘inside’ an M-2-brane, a known 1/4-supersymmetric ‘intersection’. In the latter case P represents the intersection of a 9-dimensional object with the M-2-brane; it must be interpreted as a charge associated to the boundary of an M-2-brane on an M-9-brane. The fact that P is null is an indication that the string boundary is chiral, consistent with its HW-interpretation as the heterotic string. We have now found the additional M2-brane intersections

$$(1|M2,W), \quad (1|M2,M9), \quad (10)$$

and these complete the list of possible 1/4-supersymmetric intersections involving M-2-branes.

2.3. M-5-brane

The M-5-brane worldvolume algebra is given in (Eq. (2)). We have already explained how the Y -charge leads to the intersections

$$(1|M5,M2), \quad (1|M5,M5), \quad (5|M5,KK), \quad (5|M5,M9). \quad (11)$$

Consider now the self-dual 3-form charge Z carried by a 3-brane in the M-5-brane. This charge is in the **10** representation of $Sp(2)$, which may be interpreted as either a 2-form or a 3-form in the transverse 5-space. In the 2-form interpretation, the 3-brane is the intersection with a second M-5-brane. In the 3-form interpretation, it is the intersection with an M-KK-monopole. We thus deduce the intersections

$$(3|M5,M5), \quad (3|M5,KK). \quad (12)$$

Finally, we must consider the $SU^*(4) \cong Spin(5)$ 5-vector P . If P is assumed to be a 0-form in the transverse space then P has the obvious interpretation as the null 5-momentum of an M-wave in the M-5-brane. However, we could also suppose P to be a transverse 5-form, which would indicate a 6-dimensional object. This must be interpreted as an M-KK-monopole intersecting the M-5-brane on a chiral string. We thus deduce the intersections

$$(1|M5,W), \quad (1|M5,KK). \quad (13)$$

Note that if the non-compact hyper-Kähler 4-metric of the KK-monopole were replaced by a compact hyper-Kähler metric on K_3 then the ‘intersection (1|M5,KK) would become an M-5-brane wrapped on K_3 , alias the heterotic string [23]. This goes some way to explaining why the string intersection in (1|M5,KK) is chiral. In any case, we have now found the complete set of possible M-5-brane intersections preserving 1/4 supersymmetry.

2.4. M-KK-monopole

The M-KK-monopole algebra is

$$\begin{aligned} \{Q_\alpha^i, Q_\beta^j\} = & \varepsilon^{ij} (C\Gamma_\mu)_{\alpha\beta} P^\mu + C_{\alpha\beta} Z^{(ij)} \\ & + \varepsilon^{ij} (C\Gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu} + (C\Gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho}^{(ij)}, \end{aligned} \quad (14)$$

where $i = 1, 2$ is an $Sp(1) \cong SU(2)$ index. We therefore have the following p -brane charges with their $SU(2)$ representations

$$\begin{aligned} Z(p=0):3 \quad Z(p=2):1 \quad Z(p=3):3 \\ Z(p=4):3 \quad Z(p=5):1 \end{aligned} \quad (15)$$

The $p = 4, 5$ cases come from the time components of the 3-form and 2-form charges. The algebra (Eq. (14)) is also the algebra of the D-6-brane and it will be convenient to begin with an analysis of its implications for D-6-brane intersections. In this case the transverse space is three-dimensional, so an $SU(2)$ singlet can be interpreted as either a 0-form or as a 3-form in the transverse 3-space. Similarly, an $SU(2)$ triplet can be interpreted as either a 1-form or as a 2-form in the transverse space. Each possibility yields a distinct configuration of intersecting branes. The details are much as before and the result is that one finds all 1/4-supersymmetric intersections of D-6-branes for which the intersection is a p -brane with $p = 0, 2, 3, 4, 5$. There remain two $p = 1$ intersections not yet accounted for. One is a IIA-wave in a D-6-brane which is obviously associated with P , interpreted as the null 7-momentum of the wave. The other is found by taking the space components of the null 7-vector P to be a 3-form in transverse 3-space. In this case P can be interpreted as the charge of a chiral string formed by the intersection of a D-4-brane

with the D-6-brane. This can be represented by the array

$$\begin{array}{cccccccccc} D6: & 1 & 2 & 3 & 4 & 5 & 6 & - & - & - \\ D4: & 1 & - & - & - & - & - & 7 & 8 & 9 \end{array}$$

Note that there are no dimensions in which the D-4-brane could be separated from the D-6-brane, so that a chiral theory on the intersection is indeed possible (the 11th dimension does not invalidate this argument because the M-KK-monopole is not strictly a brane with a localized worldvolume). In fact, this configuration is dual (at least formally) to a D-string in a D-9-brane, alias the (chiral) type I superstring.

Let us now return to the M-KK-monopole. The transverse space is now 4-dimensional, despite the $Spin(3)$ R-symmetry. This means that the transverse dual to, say, a transverse 1-form is now a 3-form, whereas it was previously a 2-form. However, what was previously a 1-form might now really be a two-form, in which case the transverse dual would still be a 2-form. In other words, in associating forms of two possible ranks to each worldvolume p -form in the algebra, one of the two ranks will now increase by one, the other one being unaffected. There is therefore an additional choice that must now be made, which amounts to a choice of where to put the extra dimension when re-interpreting the previous D-6-brane results in $D = 11$. The choice is not arbitrary, however, because it is determined by the possible M-branes. Only one choice has a $D = 11$ interpretation because there is only one way to interpret each D-6-brane intersection as a KK-monopole intersection. In the case of 0-brane intersections, the two D-6-brane cases lift to the intersections

$$(0|M2, KK), \quad (0|\underline{M2}, KK), \quad (16)$$

where the underlining in the second case indicates that the compact isometry direction of the KK-monopole is also an isometry of the M2-brane; in other words, the M-2-brane is wrapped on the KK circle. The two D-6-brane cases with 2-brane intersections lift to

$$(2|M2, KK), \quad (0|KK, KK). \quad (17)$$

The two 3-brane intersections are

$$(3|M5, KK), \quad (3|\underline{M5}, KK), \quad (18)$$

with the same meaning to underlining as before. Similarly, the 4-brane intersections are

$$(4|KK, KK)^a, \quad (4|KK, KK)^b, \quad (19)$$

where the notation is that of [21]: the ‘a’ case is the one in which the KK circle is common to both KK ‘monopoles’ (the arrays and supergravity solutions may be found in [21]). The 5-brane intersections are

$$(5|M5, KK), \quad (5|M9, KK). \quad (20)$$

We thus recover from the M-KK algebra the M-5-brane intersection with an M-KK-monopole deduced previously from the M-5 algebra. In addition we find the KK intersection with an M-9-brane [20]; this can be interpreted as a KK-monopole having an M-5-brane boundary on an M-9-brane.

Finally, the wave/chiral string intersections in $D = 11$ are

$$(1|W, KK), \quad (1|M5, KK), \quad (21)$$

which we deduced earlier from the M-W and M-5 algebras.

2.5. M-9-brane

The M-9-brane worldvolume algebra is

$$\begin{aligned} \{Q_\alpha, Q_\beta\} &= (C\mathcal{P}^+ \Gamma_\mu)_{\alpha\beta} P^\mu \\ &+ (C\Gamma^{\mu\nu\rho\sigma\lambda})_{\alpha\beta} Z_{\mu\nu\rho\sigma\lambda}^+, \end{aligned} \quad (22)$$

where \mathcal{P}^+ is the chiral projection operator on spinors and Z^+ is a self-dual 5-form. When this is interpreted as the D-9-brane algebra the 5-form charge is associated with a 5-brane in the D-9-brane, alias the Type I D-5-brane. We shall consider this interpretation in more detail in the following section. For its M-9-brane interpretation we must suppose that there is a one-dimensional transverse space. The 5-form Z^+ could represent either a 0-form or a 1-form in this transverse ‘1-space’. In the 0-form case we have an M-5-brane inside an M-9-brane, alias the heterotic 5-brane. In the 1-form case we have what might appear to be a 5-brane intersection of the M-9-brane by a 6-brane, but this should be interpreted as an M-KK-monopole with a boundary on the M-9-brane. Thus, we confirm the cases

$$(5|M9, M5), \quad (5|M9, KK), \quad (23)$$

found earlier from the M-5 and M-KK algebras. To find the remaining M-9-brane intersections we note that the space components of the 10-momentum P could be either 0-forms or 1-forms in the transverse space. In the former case we have the obvious interpretation as an M-wave in an M-9-brane. In the latter case the object intersecting the M-9-brane is two-dimensional; it must be an M-2-brane with boundary on the M-9-brane. The fact that P is null is again an indication of the chirality of the string boundary, which is in fact the heterotic string. Thus we deduce the intersections

$$(1|M9,W), (1|M9,M2), \quad (24)$$

which we also found previously from the M-W and M-2 algebras. We have now found the complete set of M-9-brane intersections preserving 1/4-supersymmetry.

3. Triple intersections

We now turn to triple orthogonal intersections of M-branes or Type II branes preserving 1/8 supersymmetry. The common-intersection worldvolume field theory realizes a supersymmetry algebra with 8 supercharges. The list of such algebras is as follows:

	$D = 1$	$N = 8$	$so(8)$	$\mathbf{8}_s$	
*	$D = 2$	$(8,0)$	$so(8)$	$\mathbf{8}_s$	IB-1
	$D = 2$	$(4,4)$	$so(4)$	$\mathbf{4} \oplus \mathbf{4}$	
	$D = 3$	$N = 4$	$so(4)$	$\mathbf{4}$	m-2
	$D = 4$	$N = 2$	$u(2)$	$\mathbf{2} \oplus \bar{\mathbf{2}}$	
	$D = 5$	$N = 1$	$su(2)$	$\mathbf{2}$	
*	$D = 6$	$(1,0)$	$su(2)$	$\mathbf{2}$	1B-5

The algebras marked with asterisks are those which are not the dimensional reduction of any other. These are the worldvolume algebras of the Type 1 string and 5-brane, which we call the IB-string and IB-5-brane (because of the IIB origin of the Type 1 string). Double intersections of these 1B-branes, preserving 1/4 of the supersymmetry of the Type 1 vacuum, can be considered as special cases of 1/8-supersymmetric triple intersections of IIB branes in which the ‘third brane’ is the D-9-brane. We shall consider these cases first.

The Type 1 spacetime supersymmetry algebra is the same as the M-9-brane worldvolume algebra considered above. As in that case, the null 10-vector P does ‘double duty’ as a wave momentum and chiral string charge, while Z^+ does ‘double duty’ as the charge of a 5-brane and a KK-monopole. In the past, the necessity for charges to do ‘double duty’ in the $N = 1$ $D = 10$ supersymmetry algebra has always seemed a drawback of the approach but we have just seen that it has a natural explanation from the M-9-brane perspective (as a result of the 0-form and 1-form interpretations in transverse space). In any case, we may now attempt to determine the intersection of Type 1 branes by considering the supersymmetry algebras of the 1B-string/1B-wave and the 1B-fivebrane/1B-KK-monopole. For simplicity we restrict the discussion to the 1B string and 5-brane. The 1B-string worldsheet supertranslation algebra is the $D = 2$ (8,0) algebra ($i = 1, \dots, 8$)

$$\{Q_-^i, Q_-^j\} = \delta^{ij} P_- + Z_-^{(ij)} (\mathbf{35}^+),$$

$$\delta_{ij} Z_-^{(ij)} = 0. \quad (25)$$

As indicated, the 1-form Z-charges are in the $\mathbf{35}^+$ representation of $SO(8)$, which defines a 4-form in the transverse 8-space. We conclude, correctly, that the 1B-string inside a 1B-5-brane preserves 1/4 of the supersymmetry of the Type 1 $D = 10$ Minkowski vacuum. We should be able to see this intersection from the 5-brane’s worldvolume supersymmetry algebra. This is the $D = 6$ (1,0) algebra ($i = 1, 2; \alpha = 1, \dots, 4$)

$$\{Q_\alpha^i, Q_\beta^j\} = \varepsilon^{ij} P_{[\alpha\beta]} + Z_{(\alpha\beta)}^{(ij)}. \quad (26)$$

The Z-charge is an $SU(2)$ -triplet of self-dual $D = 6$ 3-forms. This is the charge of a 3-brane in the 5-brane. The $SU(2) \cong Spin(3)$ R-symmetry of the algebra (26) suggests a spacetime interpretation in $D = 9$, but the $SU(2)$ doublet of supercharges can equally well be interpreted as a spinor of $SO(4)$, in which case an $SU(2)$ triplet is to be interpreted as a self-dual 2-form in a transverse 4-space. Thus, the 3-brane is the intersection of two 1B-5-branes. This possibility is to be expected from the IIB origin of the 1B string. We have still to find the 1B-string in the 5-brane. Since the string is chiral it should be associated with a null 6-vector. The only possibility

is P , which therefore does ‘double duty’ as a string charge and the 6-momentum of a 1B-wave in the 1B-5-brane.

Another class of 1/8-supersymmetric triple intersections is obtained by considering double intersections of branes confined to the core of a KK-monopole. For example, consider a IIB-3-brane in a IIB-KK-monopole. The IIB-3-brane is effectively confined to a $D = 6$ spacetime. In fact, this is one realization of the iib-3-brane of the ‘little’ $D = 6$ IIB string theory, alias iib-string theory³. The worldvolume supersymmetry algebra of the iib-3-brane is the $N = 2$ $D = 4$ algebra. The $\{Q, Q\}$ anticommutator, in Weyl spinor notation, is

$$\begin{aligned} \{Q_\alpha^i, Q_{\dot{\beta}j}\} &= \delta_j^i P_{\alpha\dot{\beta}} + W_{\alpha\dot{\beta}}^i{}^j, \quad W_{\alpha\dot{\beta}}^i{}^i = 0, \\ \{Q_\alpha^i, Q_\beta^j\} &= \varepsilon_{\alpha\beta} \varepsilon^{ij} Z + Y_{(\alpha\beta)}^{(ij)}. \end{aligned} \quad (27)$$

One sees from this that the iib-3-brane admits 1/2-supersymmetric p -branes for $p = 0, 1, 2, 3$. The R-symmetry group is $U(2)$. We interpret the $U(1) \cong Spin(2)$ factor as the rotation group of the 2-space transverse to the worldvolume of the $D = 6$ iib-3-brane, and the $SU(2)$ factor as the rotational isometry group of the KK-monopole implicit in the iib-theory’s $D = 10$ spacetime realization. The iib- p -brane charges are in the following $U(2)$ representations:

$$\begin{aligned} Z(p=0): \mathbf{1}_{+1} \quad W(p=1): \mathbf{3}_0 \quad Y(p=2): \mathbf{3}_{+1} \\ W(p=3): \mathbf{3}_0 \end{aligned} \quad (28)$$

where the subscript on the $SU(2)$ representations indicates the $U(1)$ charge (Z and Y are complex). The 0-branes must be interpreted as intersections, or endpoints, of iib-strings on iib-3-branes. Thus iib-3-branes are d-branes for iib-strings, a fact that is evident from their $D = 10$ spacetime interpretation as IIB-branes in a KK-monopole. The 1-branes are intersections with a IIB-3-brane for which the two orthogonal directions span a 2-cycle in the 4-space of the KK-monopole (this is determined by a 2-form in 3-space if the IIB brane is not wrapped on the KK circle and by a 1-form in 3-space if it is wrapped on the KK circle). The 2-branes can be interpreted as

intersections with other $D = 6$ 3-branes that are actually IIB-5-branes with some directions in the 4-space of the KK-monopole. The 3-brane can be interpreted as the original iib-3-brane inside a IIB-5-brane for which two directions are in the KK 4-space.

Another interpretation of the iib-string and iib-3-brane is as the string and 3-brane in the M-5-brane, so the intersections of iib-branes just discussed have implications for triple M-brane intersections. For example, we saw that two iib-3-branes can intersect on a string. We conclude that the same must be true of two 3-branes in an M-5-brane, as it is because it is implied by the following triple intersection of M-5-branes preserving 1/8 supersymmetry [10]

$$\begin{array}{llllllllll} \text{M-5:} & 1 & 2 & 3 & 4 & 5 & - & - & - & - & - \\ \text{M-5:} & 1 & 2 & 3 & - & - & 6 & 7 & - & - & - \\ \text{M-5:} & 1 & - & - & 4 & 5 & 6 & 7 & - & - & - \end{array}$$

The 3-branes in M-5-branes can also intersect on a 2-brane because this is implied by the alternative triple M-5-brane intersection

$$\begin{array}{llllllllll} \text{M-2:} & 1 & 2 & 3 & 4 & 5 & - & - & - & - & - \\ \text{M-5:} & 1 & 2 & 3 & - & - & 6 & 7 & - & - & - \\ \text{M-5:} & 1 & 2 & - & - & 5 & 6 & - & - & - & - \end{array}$$

The $p = 3$ case of (Eq. (28)) is realized by the triple M-5-brane intersection

$$\begin{array}{llllllllll} \text{M-2:} & 1 & 2 & 3 & 4 & 5 & - & - & - & - & - \\ \text{M-5:} & 1 & 2 & 3 & - & - & 6 & 7 & - & - & - \\ \text{M-5:} & 1 & 2 & 3 & - & - & - & - & 8 & 9 & - \end{array}$$

The 3-brane intersection of the first two M-5-branes lies inside the third M-5-brane.

This leaves only the $p = 0$ case of (Eq. (28)) to be accounted for as a triple M-brane intersection. We have already deduced that it must be an endpoint on a 3-brane of the iib-string, now in its guise as the self-dual string in the M-5-brane. The fact that the iib string can end on the iib 3-brane can now be seen from the triple M-brane intersection array

$$\begin{array}{llllllllll} \text{M-5:} & 1 & 2 & 3 & 4 & 5 & - & - & - & - & - \\ \text{M-5:} & 1 & 2 & 3 & - & - & 6 & 7 & - & - & - \\ \text{M-2:} & - & - & - & - & 5 & 6 & - & - & - & - \end{array}$$

This can be interpreted as the intersection of two M-5-branes on a 3-brane with the 3-brane intersecting the string boundary of an M-2-brane. The membrane boundary is continuous but switches from one M-5-brane to the other at the intersection point. From the perspective of one of the M-5-branes the string boundary appears to end on a 3-brane.

³ We have found the discussion of ‘little’ theories in [22] useful in orienting our ideas. We refer to that paper for further references.

4. Discussion

We have seen that $1/4$ supersymmetric intersections of M-branes, and Type II superstring branes, are encoded in the supersymmetry algebras with 16 supercharges interpreted as worldvolume supersymmetry algebras. The interpretation of the M-brane algebras is particularly natural. We have also shown how some algebras with 8 supercharges encode triple intersections, but the interpretation is less natural in the sense that there are various ambiguities that arise which must be resolved by additional information. This is probably inevitable because a $1/8$ supersymmetric M-brane configuration, to take an example, could be a non-orthogonal double intersection rather than an orthogonal triple intersection. Thus, it is likely that the algebras with less than 16 supercharges have several inequivalent interpretations as intersection worldvolume algebras. However, we suspect that there exists some natural iteration procedure that will at least allow a classification of all orthogonal multiple intersections from the supersymmetry algebras with 16, 8, 4, 2, 1 supercharges. In any case, it is now clear that the various supersymmetry algebras in various dimensions encode an enormous amount of information about partially supersymmetric configurations of branes in M-theory.

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